
Submit the solutions in groups of two at the lecture on Tuesday, 2018-06-26

Exercise 1. (a) Suppose ϕ is of class C^2 in an interval $[a, b]$. Assume that $|\phi'(t)| \geq 1$ for all $t \in [a, b]$ and that ϕ' is monotone. Prove

$$\left| \int_a^b e^{i\lambda\phi(t)} dt \right| \leq c\lambda^{-1}$$

for all $\lambda > 0$ and c independent of a and b . (Hint: integrate by parts)

(b) Show that the conclusion may fail if ϕ' is not monotone.

Exercise 2. Consider the Hausdorff–Young inequality on real line

$$\|\widehat{f}\|_q \leq C_p \|f\|_p$$

for Schwartz functions. Show that the conditions $p \leq 2$ and $q = p/(p-1)$ are necessary for it to be true. (Hint: try $e^{-s|x|^2}$ with $s \in 1 + i\mathbb{R}$)