CLASSIFICATION OF ADDITIVE GROUPS
- ABSTRACT -

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The title of this talk is, while catchy, somewhat of a white lie. We will spend most of the time introducing the basic notion of a *group scheme*. No knowledge of scheme theory is required however, since we will focus for the most part on the case of affine (commutative) group schemes over some field k (aka *algebraic groups*). These are just k-algebras with some extra structure. In particular, they have both multiplication and **co**multiplication,

$$B \otimes_k B \to B \text{ and } B \to B \otimes_k B.$$

The most intuitive way to think of a group scheme G over k is as a representable functor

$$G \colon \{k\text{-algebras}\} \to \{\text{groups}\}$$

via the Yoneda embedding (which I will explain, depending on the audience's preference). The fact that G lands in the category of groups rather than sets is what makes it a group object, and gives it the aforementioned extra structure. It makes a lot of definitions rather straight-forward, e.g. G is commutative if its target is the category of abelian groups.
I will give a plethora of examples of these objects. In particular, we will see that group schemes are a much more refined notion than groups. Even over an algebraically closed field k, not every group scheme is "constant" (i.e. coming from a group).
In fact, in the second half of the talk, we will concentrate on the case where k has positive characteristic. The goal is to sketch the classification of commutative group schemes "of additive type" over k.
Namely, we can translate them into a (semi-)linear algebraic setting via their Dieudonné modules. One great feature of characteristic p is the existence of the Frobenius morphism F (and – as it turns out – its "dual" V). The Dieudonné ring is defined in terms of these, and restricting to additive type means $V = 0$. What remains is to study modules over a polynomial ring in F. This results in the following classification theorem:

Theorem 0.1. *Every commutative group scheme of additive type over an algebraically closed field of characteristic p is a product of group schemes of the form $\mathbb{G}_a$, α_{p^r} and $\underline{\mathbb{Z}/p}$.*

Unfortunately, most applications of group schemes are beyond the scope of this talk. Nonetheless, I will try to motivate their study by giving a rundown of where they appear and what they have been used for.

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