Minisymposium 3 - Stochastic Processes with Jumps: Theory and applications

Optimal Series Representation of Certain Gaussian Processes

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Let (T, ρ) be a compact metric space and let $X = (X(t), t \in T)$ be a centered Gaussian process over T possessing a.s. continuous paths. Then there are continuous functions u_k from T into $\mathbb{R}$ such that a.s.

$$X(t) = \sum_{k=1}^{\infty} \xi_k u_k(t) , \qquad t \in T .$$

Here $(\xi_k)_{k\geq 1}$ denotes an i.i.d. sequence of $\mathcal{N}(0,1)$–distributed random variables. Since this series representation of X is not unique, it is naturally to ask for optimal ones, i.e., for those where

$$\left(\mathbb{E} \sup_{t\in T} \left| \sum_{k=n}^{\infty} \xi_k u_k(t) \right|^2 \right)^{1/2} ,$$

as $n \to \infty$, tends to zero as fast as possible.

We investigate this problem for the fractional Brownian sheet on $[0,1]^N$ and for Lévy's fractional Brownian motion over a self–similar set $T \subset \mathbb{R}^N$. Optimal representations are obtained via suitable wavelet decompositions. The presented results rest on joint works with Thomas Kühn from Leipzig and with Antoine Ayache from Lille.