Minisymposium 5 - Finanznumerik (Computational Finance)

Nonlinear Black-Scholes-type Equations for Financial Derivatives

Ansgar Jüngel (Universität Mainz)

Standard financial derivatives like European options are priced by the famous Black-Scholes model which has the form of a linear parabolic equation. The Black-Scholes equation is derived under quite restrictive assumptions, e.g., no transaction costs occur and the market is complete. Without these conditions the resulting models may become nonlinear due to feedback effects, for instance.

In this talk some nonlinear Black-Scholes-type equations are discussed. The first model including the effect of transaction costs is a Black-Scholes equation with a volatility depending on the second derivatives of the solution. It has been derived by Barles and Soner in 1998. The equation is discretized using a higher order compact finite difference scheme and some numerical convergence results are given. The solutions are compared to those from the standard Black-Scholes model.

The second model describes the optimal value function in incomplete markets and has been derived by Leitner in 2001. It gives information on the transaction of shares the investor should make in order to maximise her or his profit. The market is allowed to possess non-tradable state variables like the employee income, weather parameter etc. Mathematically, the model is a parabolic equation with quadratic gradients. Existence and uniqueness results and numerical simulations are presented in order to show the influence of the nontradable state variables.