Minisymposium 5 - Finanznumerik (Computational Finance)

Fast and Accurate Pricing of Early Exercise Options with the Fast Fourier Transform

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When valuing and risk managing exotic derivatives, practitioners demand fast and accurate prices and sensitivities. Since the models being used in practice are becoming increasingly more complex, efficient methods have to be developed that can cope with the high dimensionality of such models. Aside from non-standard exotic derivatives, plain vanilla options in many stock markets are actually of the American type. As any pricing and risk management system has to be able to calibrate to these plain vanilla options, it is of the utmost importance to be able to value these American options quickly and accurately. In this paper we present highly efficient methods for options with early exercise or callable features. The methods developed rely heavily on the fast Fourier Transform (FFT). Early FFT based solution methods were mainly developed for pricing European options. Andricopoulos introduced the quadrature method QUAD, which can be used to value a variety of options with exotic features, assuming that the underlying process follows geometric Brownian motion. OSullivan combined the early Fourier transform methods with the QUAD method, and enabled the use of a fast resolution of the probability density function of the logarithm of the underlying. His method can be applied to a general set of underlying Levy processes as well as to exotic path dependent features. Inspired by the quadrature pricing techniques, we propose a solution method that utilises the power of the FFT as much as possible. At the same time, we wish to get a grip on the resulting errors. The computational complexity in our recently developed solution methods reduces to $O(MN \log N)$, with M the number of observation times, and N the number of points in price dimension. In combination with Richardson extrapolation, and by using the fractional FFT method, we aim for the highest efficiency for the pricing of callable options under models where the underlying asset is exponentially affine in the state variables. This is the case for exponentially affine jump-diffusion models, exponential Levy models, and variants hereof. We focus on both one-dimensional and two-dimensional processes, the latter class allowing us to develop efficient pricing methods for popular stochastic volatility models, such as the Heston model. The methods proposed are compared, also with iterative solution methods that solve the partial (integro-) differential equation with finite differences for a variety of American and Bermudan option pricing models with early exercise features.