Minisymposium 16 - Set Theory

Co-stationarity of the ground model

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The bulk of this talk is based on joint work with Sy-David Friedman. Given $V \subseteq W$ models of ZFC with the same ordinals and $\kappa < \lambda$ cardinals in W with κ regular, let $\mathcal{P}_\kappa(\lambda)$ denote the collection of subsets of λ of size less than κ in W. We say that the ground model is *co-stationary* if $\mathcal{P}_\kappa(\lambda) \setminus V$ is stationary in $\mathcal{P}_\kappa(\lambda)$. Gitik showed the following: Suppose κ is a regular cardinal in W, and λ is greater than or equal to $(\kappa^+)^W$. If there is a real in $W \setminus V$, then the ground model is co-stationary in $\mathcal{P}_\kappa(\lambda)$.
We consider problems of generalizing Gitik's Theorem to forcing extensions in which no reals are added. In particular, we show that the analogue of Gitik's Theorem for $\aleph_2$-c.c. forcings which add a new subset of $\aleph_1$ (but no new ω-sequences) is equiconsistent with a class of Erdös cardinals. The necessity of ω_1-Erdös cardinals follows from a covering theorem of Magidor. For regular $\kappa \geq \aleph_2$ with $\aleph_\kappa > \kappa$, the co-stationarity of the ground model in the $\mathcal{P}_{\kappa^+}(\aleph_\kappa)$ of a κ-Cohen forcing extension is equiconsistent with κ measureable cardinals.
For $\nu \geq \aleph_1$ we present some consistency results concerning partial orderings which add a new ν-sequence but no new subset of ν. We also include some more recent work with Justin Moore concerning partial orderings which add a new ω-sequence without adding a new real.