Minisymposium 17 - Globale Analysis

Characterization of weak boundary values of L^p-functions by approximation

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Let $D \subset \mathbb{C}^n$ be bounded with $C1$-boundary and $1 \leq p < \infty$. Then $f \in L^p(D)$ with $df \in L^p_1(D)$ has boundary values $f_b \in L^p(\partial D)$ such that Stokes Theorem is valid. If we just know $\overline{\partial} f \in L^p_{0,1}(D)$ we say that f has boundary values $f_b \in L^p(\partial D)$ if the Stokes Formula

(1) $$\int_{\partial D} f_b \phi|_{\partial D} = \int_D \overline{\partial} f \wedge \phi + \int_D f \overline{\partial} \phi$$

holds for all $\phi \in C^\infty(\overline{D})$. Such boundary values play a decisive role in the study of the boundary regularity of the $\overline{\partial}$-equation or the complex Green operator. In this talk we show that the space of functions with such L^p-boundary values is exactly the completition of $C^\infty(\overline{D})$ under the norm

$$\|f\|_* = \|f\|_{L^p(D)} + \|\overline{\partial} f\|_{L^p_{0,1}(D)} + \|f|_{\partial D}\|_{L^p(\partial D)}.$$

Similar results are true for forms of higher degree. As applications, we show that $f \in L1_{loc}(D)$ with $\overline{\partial} f = 0$ in the sense of distributions is C^∞-smooth and that Stokes Formula (1) holds for $f \in C0(\overline{D})$ with $\overline{\partial} f \in L1_{0,1}(D)$ (in that case $f_b = f|_{\partial D}$).