Minisymposium 16

Set Theory

Leiter des Symposiums:

Prof. Dr. Ernest Schimmerling
Dept. of Mathematical Sciences

Carnegie Mellon University
Pittsburgh, PA 15213-3890, USA

Prof. Dr. Ralf Schindler
Institut für Mathematische Logik
und Grundlagenforschung
Universität Münster
Einsteinstr. 62
48145 Münster, Germany

Set theory originally grew out of analysis and topology, and was later influenced by combinatorics. The minisymposium lectures reflect these roots. What distinguishes set theory from these other fields is its strong connection to logic, which also figures prominently in the topics and techniques represented in the talks. In fact, each of the results to be discussed has dual appeal, applied and foundational.

Donnerstag, 21. September

Kleiner Hörsaal, Mathematisches Institut, Wegelerstr. 10

15:00 – 15:50 **Matteo Viale** *(Torino)*
Applications of the Proper Forcing Axiom to cardinal arithmetic

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16:00 – 16:50 **Boban Veličkoviċ** *(Paris)*
The consistency strength of a five element basis for uncountable linear orderings

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17:00 – 17:20 **Jordi Lopez-Abad** *(Paris)*
Sequences on Banach spaces and finite sets of integers

17:30 – 17:50 **Ilijas Farah** *(Toronto)*
Submeasures, Lévy groups and extremely amenable groups

Freitag, 22. September

Kleiner Hörsaal, Mathematisches Institut, Wegelerstr. 10

15:00 – 15:20 **Gunter Fuchs** *(Münster)*
Degrees of Rigidity for Souslin Trees and Changing the Heights of Automorphism Towers

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15:30 – 15:50 **David Aspero** *(Bristol)*
Definable well–orders of $H(\omega_2)$ and forcing axioms

16:00 – 16:20 **Natasha Dobrinen** *(Wien)*
Co-stationarity of the ground model

16:30 – 16:50 **Otmar Spinas** *(Kiel)*
Perfect Set Theorems

17:00 – 17:50 **Jouko Väänänen** *(Helsinki)*
Strong Logics

Vortragsauszüge

Matteo Viale *(Torino)*
Applications of the Proper Forcing Axiom to cardinal arithmetic

I will survey the effects of the Proper Forcing Axiom on cardinal arithmetic.

Boban Veličkoviȁ *(Paris)*
The consistency strength of a five element basis for uncountable linear orderings

Moore showed in 2004 that the Proper Forcing Axiom implies that there is a five element basis for the class of uncountable linear orders, thus confirming a well known conjecture of Shelah. The assumptions needed in the original proof have consistency strength of at least infinitely many Woodin cardinals. In this talk we will show that the upper bound on the consistency strength of the existence of such a basis is less than a Mahlo cardinal, a hypothesis which can hold in the constructible universe. The key notion in our proof is the saturation of an Aronszajn tree, a concept which was studied by Baumgartner in the 1970s. In particular, we show that the saturation of an Aronszajn tree together with the Bounded Proper Forcing Axiom suffices for the existence of a five element linear basis.

This is joint work with B. Koenig, P. Larson and J. Moore.

Jordi Lopez-Abad *(Paris)*
Sequences on Banach spaces and finite sets of integers

The purpose of this talk is to present a framework for studying the structure of subsequences of a given infinite sequence in a real Banach space X using combinatorics of finite sets of integers. We focus mainly on weakly-null sequences. The main combinatorial tool is the notion of barrier introduced by Nash-Williams.

Ilijas Farah *(Toronto)*

Submeasures, Lévy groups and extremely amenable groups

Given a locally compact group H and a submeasure ϕ on clopen subsets of 2^N, consider the group $S(\phi, H)$ of simple H-valued functions with respect to the topology of convergence in ϕ. These groups can be used to distinguish some of the standard classes of 'massive' topological groups.

This is a joint work with S. Solecki.

Gunter Fuchs *(Münster)*

Degrees of Rigidity for Souslin Trees and Changing the Heights of Automorphism Towers

Various strong notions of rigidity for Souslin trees are investigated and separated, assuming the diamond principle, into a hierarchy. Most of these rigidity properties state that a tree has a certain rigidity property in any model obtained by forcing with the tree itself.

An application to the automorphism tower problem is given, showing that, again assuming diamond, there is a group the height of whose automorphism tower is highly malleable by forcing with certain Souslin trees. Carrying out the construction at higher cardinality levels gives the full statement on changing the heights of automorphism towers, that was realized by Hamkins and Thomas using proper class forcing, in L.

David Aspero *(Bristol)*

Definable well–orders of $H(\omega_2)$ and forcing axioms

This talk deals with the problem of building set-forcing extensions in which there is a simple definition, over the structure $\langle H(\omega_2), \in\rangle$ and without parameters, of a prescribed member of $H(\omega_2)$ or of a well–order of $H(\omega_2)$, possibly together with some strong forcing axiom.

I will present two theorems. The first one is an optimal result, with respect to the logical complexity of the definitions involved, at the level of the structure $\langle H(\omega_2), \in, NS_{\omega_1}\rangle$. This

result is a particular case of a much more general theorem applying to $H(\kappa^+)$ for every uncountable regular cardinal κ.
The second theorem I will present says that, under the assumption that there is a supercompact cardinal, there is a partial order forcing both the existence of a well–order of $H(\omega_2)$ definable, over $\langle H(\omega_2), \in\rangle$, by a formula without parameters, and that the forcing axiom PFA^{++} holds.

Natasha Dobrinen *(Wien)*
Co-stationarity of the ground model

The bulk of this talk is based on joint work with Sy-David Friedman. Given $V \subseteq W$ models of ZFC with the same ordinals and $\kappa < \lambda$ cardinals in W with κ regular, let $\mathcal{P}_\kappa(\lambda)$ denote the collection of subsets of λ of size less than κ in W. We say that the ground model is *co-stationary* if $\mathcal{P}_\kappa(\lambda) \setminus V$ is stationary in $\mathcal{P}_\kappa(\lambda)$. Gitik showed the following: Suppose κ is a regular cardinal in W, and λ is greater than or equal to $(\kappa^+)^W$. If there is a real in $W \setminus V$, then the ground model is co-stationary in $\mathcal{P}_\kappa(\lambda)$.
We consider problems of generalizing Gitik's Theorem to forcing extensions in which no reals are added. In particular, we show that the analogue of Gitik's Theorem for $\aleph_2$-c.c. forcings which add a new subset of $\aleph_1$ (but no new ω-sequences) is equiconsistent with a class of Erdös cardinals. The necessity of ω_1-Erdös cardinals follows from a covering theorem of Magidor. For regular $\kappa \geq \aleph_2$ with $\aleph_\kappa > \kappa$, the co-stationarity of the ground model in the $\mathcal{P}_{\kappa^+}(\aleph_\kappa)$ of a κ-Cohen forcing extension is equiconsistent with κ measureable cardinals.
For $\nu \geq \aleph_1$ we present some consistency results concerning partial orderings which add a new ν-sequence but no new subset of ν. We also include some more recent work with Justin Moore concerning partial orderings which add a new ω-sequence without adding a new real.

Otmar Spinas *(Kiel)*
Perfect Set Theorems

I will present several results and open problems in the context of searching for perfect set theorems for the following largeness conditions for subsets of Cantor or Baire space: splitting property, refining property, infinitely often equal property.

Jouko Väänänen *(Helsinki)*

Strong Logics

I will describe strong logics that arise naturally in database theory. I will discuss set theoretic questions related to their model theory in the infinite context. In particular, I will talk about recent joint work with Magidor on Löwenheim-Skolem type properties of strong logics.