Minisymposium 7

Stochastic algorithms and Markov processes

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Markov chain Monte Carlo methods, sequential Monte Carlo methods, and other related stochastic algorithms have become widely used tools in many application fields of mathematics. Despite their massive use, the theoretical understanding of convergence properties of these algorithms is often rather rudimentary. This is in particular the case in high dimensional models that typically arise in many applications. Whereas formerly, research has often been carried out more or less independently in probability theory, stochastic analysis, and statistical mechanics on the one side, and theoretical computer science, discrete mathematics, and numerical analysis on the other side, recently there is a rapidly growing activity at the borderline of the different disciplines. Besides classical probabilistic techniques (*e.g.* martingale methods), concepts from statistical mechanics (phase transitions, critical slowing down), and techniques from stochastic analysis (decay to equilibrium of Markov processes, spectral gap estimates) as well as infinite dimensional analysis (*e.g.* logarithmic Sobolev inequalities) have become crucial.

Montag, 18. September

HS XV, Hauptgebäude, Regina-Pacis-Weg

14:30 – 15:30 **Olle Häggström** *(Göteborg)*

15:30 – 16:00 **Nikolaus Schweizer** *(Bonn)*
Local spectral gaps on the mean field Ising model and Multilevel MCMC methods

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16:00 – 16:30 **Emilio de Santis** *(Roma)*
Exact sampling for discrete time spin systems and unilateral fields

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16:30 – 17:00 *break*

17:00 – 18:00 **Pierre del Moral** *(Nice)*
Coalescent tree based functional representations for some Feynman-Kac particle models

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18:00 – 18:30 **Andreas Eberle** *(Bonn)*
Convergence of sequential MCMC methods I

18:30 – 19:00 **Carlo Marinelli** *(Bonn)*
Convergence of sequential MCMC methods II

Dienstag, 19. September

Großer Hörsaal, Mathematisches Institut, Wegelerstr. 10

14:30 – 15:30 **Dan Crisan** *(Imperial College London)*
Solving the filtering problem in a continuous time framework. Advantages and pitfalls

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15:30 – 16:00 **Wilhelm Stannat** *(Darmstadt)*
On stability of the optimal filter

16:00 – 16:30 **Samy Tindel** *(Nancy)*
A model of Brownian directed polymer in a Gaussian random environment

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16:30 – 17:00 *break*

17:00 – 18:00 **Mark Jerrum** *(Edinburgh)*
Tight bounds on mixing time of Markov chains

18:00 – 19:00 **Marek Karpinski** *(Bonn)*
Metric construction for path coupling, rapid mixing, and approximate counting

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Mittwoch, 20. September

Großer Hörsaal, Mathematisches Institut, Wegelerstr. 10

14:30 – 15:30 **Wilfrid S. Kendall** *(Warwick)*
Perfect simulation: a survey and recent developments

15:30 – 16:30 **Marie-Colette van Lieshout** *(Amsterdam)*
Perfect simulation for length-interacting polygonal Markov fields in the plane

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16:30 – 17:00 *break*

17:00 – 18:00 **Rudolf Grübel** *(Hannover)*
Markov chains in the analysis of algorithms

18:00 – 18:30 **Ralph Neininger** *(Frankfurt))*
The size of random fragmentation trees

Vortragsauszüge

Olle Häggström *(Göteborg)*
Problem solving is often a matter of cooking up an appropriate Markov chain

By means of a series of examples, taken from classic contributions to probability theory as well as from my own practice, I will try to convince the audience of the claim made in the title of the talk. Along the way, I will have reason to discuss topics such as coupling, correlation inequalities, and percolation.

Nikolaus Schweizer *(Bonn)*
Local spectral gaps on the mean field Ising model and Multilevel MCMC methods

I consider the Metropolis Markov Chain based on the nearest neighbor random walk on the positive half of the Mean Field Ising Model, i.e., on those vectors from $\{-1, 1\}^N$ which contain more 1 than -1. Using randomly-chosen paths I prove a lower bound for the Spectral Gap of this chain which is of order N^{-2} and which does not depend on the inverse temperature β.

In conjunction with decomposition results such as those in Jerrum, Son, Tetali and Vigoda (2004) this result may be useful for bounding the spectral gaps of more complex Markov chains on the Mean Field Ising Model which may be decomposed into Metropolis chains. As an example, I apply the result to two Multilevel Markov Chain Monte Carlo algorithms, Swapping and Simulated Tempering. Improving a result by Madras and Zheng (2002), I show that the spectral gaps of both algorithms on the (full) Mean Field Ising Model are bounded below by the reciprocal of a polynomial in the lattice size N and in the inverse temperature β.

Emilio de Santis *(Roma)*
Exact sampling for discrete time spin systems and unilateral fields

We present a generalization of the technique of Häggström and Steif (2000) for the exact simulation of finite sections of infinite-volume Gibbs random fields. The main role is played by an auxiliary binary field, which indicates the sampling region. Percolation bounds can be used to prove that the algorithm terminates almost surely. In the simplest case this field is Bernoulli; however a new blocking technique can be used that destroy the independence property but extend the validity of the algorithm. A connection with stationary unilateral fields in the plane, considered by Pickard (1980) and Galbraith and Walley (1982), is discussed.

Pierre del Moral *(Nice)*
Coalescent tree based functional representations for some Feynman-Kac particle models

We present tree based functional representations of a class of Feynman-Kac particle distributions, including an extension of the Wick product formula for interacting particle systems on coalescent forests. These weak expansions rely on an original combinatorial, and permutation group analysis on a special class of coalescent, and colored forests. We also show that these polynomial type representations provide non asymptotic and sharp propagation of chaos type properties, as well as sharp L^p-mean error bounds, and laws of large numbers for U-statistics.

Andreas Eberle *(Bonn)*
Convergence of sequential MCMC methods I

We study convergence properties of a class of stochastic algorithms for Monte Carlo integral estimation w.r.t. probability distributions, which combine elements of Markov chain Monte Carlo methods and importance sampling/ resampling schemes. We develop an analysis by funtional inequalities for an associated nonlinear flow of probability measures. This allows us to prove that the combined methods are sometimes converging rapidly in multimodal setups where traditional MCMC methods mix extremely slowly. For example, we can prove rapid convergence in the mean field Ising model at all temperatures.

Carlo Marinelli *(Bonn)*

Convergence of sequential MCMC methods II

We study convergence properties of a class of stochastic algorithms for Monte Carlo integral estimation w.r.t. probability distributions, which combine elements of Markov chain Monte Carlo methods and importance sampling/ resampling schemes. We develop an analysis by funtional inequalities for an associated nonlinear flow of probability measures. This allows us to prove that the combined methods are sometimes converging rapidly in multimodal setups where traditional MCMC methods mix extremely slowly. For example, we can prove rapid convergence in the mean field Ising model at all temperatures.

Dan Crisan *(Imperial College London)*

Solving the filtering problem in a continuous time framework. Advantages and pitfalls

Particle filters have enjoyed a period of fast development in the last fifteen years both from the theoretical and from the applicative viewpoint. For many filtering problems, a natural mathematical model for the signal is a continuous time Markov process that satisfies a stochastic diferential equation of the form

$$dx_t = f(x_t)\,dt + \sigma(x_t)\,dv_t, \tag{1}$$

where v is a Wiener process whilst the observation is modelled by an evolution equation of the form

$$dy_t = h(x_t)\,dt + dw_t.$$

where w is a Wiener process independent of v.

Within the continuous time framework, $\pi = \{\pi_t, t \geq 0\}$ the conditional distribution of the signal x_t given the observation data $\{y_s, s \in [0,T]\}$ is the solution of a nonlinear stochastic PDE, called the Kushner-Stratonovitch with no explicit solution in the general case. For a suitable class of functions $\varphi,\ \pi_t(\varphi)$ can be viewed as the expected value of a certain functional *parametrized* by the observation path $\{y_s, s \in [0,T]\}$ of a process ξ which is a solution of (1). In other words, we seek to obtain something akin to what in the theory of approximation for stochastic differential equations is called a *weak solution* of (1).

This fundamental observation leads to approximating algorithms for the filtering problem obtained by adapting existing weak approximations of SDEs to the filtering framework. Firstly, one approximates π by replacing the (continuous) observation path with a discrete version. The standard method is to choose an equidistant partition $\{i\delta,\, i = 0, 1, ...\}$ of the timeline and consider only the observation data $\{y_{i\delta},\, i = 0, 1, ...\}$ corresponding to the partition time instances. The resulting probability measure π^δ converges to π as δ tends to 0. We present a number of convergence results regarding for approximations of π^δ.

Wilhelm Stannat *(Darmstadt)*
On stability of the optimal filter

Estimating a Markovian signal process observed with independent noise has many important applications not only in engineering sciences. The optimal estimate of the signal depends, of course, on the one hand on the observations but on the other hand also on the initial state of the signal. Since the signal is not observed directly, its initial state, however, is unknown. It is therefore our interest to understand the dependence of the optimal filter w.r.t. the initial state. In our talk we show how a variational approach can be used to understand this dependence and how to obtain explicit a priori lower bounds of variational type on the rate of stability. Our results are helpful for the design of efficient measurements. We will also have a closer look at a particular class of signal processes with multiplicative noise arising in the theory of software reliability.

Samy Tindel *(Nancy)*
A model of Brownian directed polymer in a Gaussian random environment

In this talk, we will study a Gibbs measure based on a model of Brownian directed polymer in a Gaussian random medium. This model is a continuous analog of some models of discrete random walks in a iid Gaussian potential, and is mainly parametrized by the spatial covariance Q of the random environment. Our hope is that the high number of methods at hand in this case (Brownian scaling, Malliavin calculus, Gaussian tools, analogy with Lyapounov exponents for SPDEs) will allow us to give a rather complete description of the Gibbs measure under consideration, and we will present here some estimates on the partition function of the model, as well as a lower bound on the growth of the polymer.

Mark Jerrum *(Edinburgh)*
Tight bounds on mixing time of Markov chains

The mixing time of a Markov chain is the time taken, starting from a fixed initial state, for it to converge to near-stationarity. Convergence is usually measured in terms of total variation distance. Good upper bounds on mixing time are required in the analysis of many randomised algorithms. Tight bounds are hard to come by, though it is well known that coupling arguments can sometimes yield results. This talk will concentrate on lesser known techniques, such as harmonic analysis or logarithmic Sobolev inequalities.

Marek Karpinski *(Bonn)*
Metric construction for path coupling, rapid mixing, and approximate counting

Coupling techniques have a long history in the theory of Markov chains, and can be used to obtain quantitative estimates of their convergence times, and rapid mixing. Good coupling are usually difficult to design, but path coupling has recently proved a very useful technique for constructing and analysing them. The basic idea here is to restrict the design of the coupling to pairs of states which are close in some suitable metric on the state space. We prove a general theorem for path coupling using stopping times based on a particular construction of metric which enables us to work with the standard one-step path coupling. We apply this result to design efficient approximation algorithms for several hard counting problems, like counting hypergraph independent sets and colorings.

Joint work with M. Bordewich and M. Dyer.

Wilfrid S. Kendall *(Warwick)*
Perfect simulation: a survey and recent developments

The technique of *exact* or *perfect simulation* was introduced in the celebrated paper of Propp and Wilson (1996), which showed how in favourable cases one might improve Markov chain Monte Carlo algorithms so as to deliver exact draws from statistical equilibrium. Since then the technique has seen much theoretical and practical development.

This talk will review ideas of perfect simulation and present an update to the survey of Kendall (2005). In particular I will discuss general results which show how one might carry out perfect simulation *in principle* (if not in practice) for general geometrically ergodic Markov chains and some generalizations.

Joint work with Stephen Connor.

References:

W. S. Kendall (2005), "Notes on Perfect Simulation" in *Markov chain Monte Carlo: Innovations and Applications* (edited by W. S. Kendall, F. Liang, J.-S. Wang); (2005) pp 93-146.
J. G. Propp and D. B. Wilson (1996), "Exact sampling with coupled Markov chains and applications to statistical mechanics", *Random Structures and Algorithms*, 9, 223-252.

Marie-Colette van Lieshout *(Amsterdam)*

Perfect simulation for length-interacting polygonal Markov fields in the plane

We construct perfect samplers for length-interacting Arak-Clifford-Surgailis polygonal Markov fields in the plane. This is achieved by providing for the polygonal fields a hard-core-interacting marked point process representation with individual points carrying polygonal loops as their marks. This enables us to use the general framework and software of (Van Lieshout and Stoica, 2006), in particular their generalised coupling-from-the-past (Kendall, 1998) and clan of ancestors (Fernandez et al, 2002) routines for our particular purposes.

Rudolf Grübel *(Hannover)*

Markov chains in the analysis of algorithms

The classical approach to the analysis of randomized algorithms or deterministic algorithms with random input concentrated on the average case behaviour, i.e. on the expectation EX_n of some random variable X_n that represents the complexity or running time of the algorithm as a function of the input size n. In the last 15-20 years considerable progress has been made in the analysis of the full distribution of X_n, and many limit results as $n \to \infty$ have been obtained for a variety of standard algorithms. In this context, Markov chains play an important role. We give several examples and discuss the general methodology.

Ralph Neininger *(Frankfurt)*

The size of random fragmentation trees

Random trees generated by a class of random fragmentation procedures are discussed with respect to their size. These trees are motivated by corresponding search trees that are frequently used in Computer Science as data structures where size corresponds to the memory needed to store the tree.
We show that the size of these trees, after normalization, is asymptotically normal for a wide class of such fragmentation procedures whereas for other fragmentation procedures we characterize their periodic behavior.

This talk is based on joint work with Svante Janson.