Problem sheet 7 Rigid analytic geometry Winter term 2024/25

As at the end of the last sheet let $(Y, \preceq)$ be a Priestley space and $\mathfrak{B}$ the set of clopen subsets $\Omega \subseteq Y$ such that $y \in \Omega$ and $y \preceq v$ implies $v \in \Omega$. Then $\mathfrak{B}$ is closed under finite intersections within Y, including the empty intersection Y. Let Y^s be Y equipped with the topology for which $\mathfrak{B}$ is a topology base. The following finishes the proof that Y^s is a spectral space.

Problem 1 (3 points)**.** *Let $Z \subseteq Y^s$ be closed and irreducible. Show that Y contains a generic point.*

Problem 2 (1 point)**.** *Show that $\mathfrak{B}$ is the set of quasicompact open subsets of Y^s.*

Problem 3 (3 points)**.** *Show that $Y \xrightarrow{\mathrm{Id}_Y} (Y^s)_{\mathrm{con}}$ is a homeomorphism.*

In the following, the results of subsection 2.1 which have been fully shown or marked as trivial with an OK-hook in the lecture can of course be used. Let R be a topological ring, M a topological R-module and $X \subseteq M$ a bounded subset.

Problem 4 (3 points)**.** *If $Y \subseteq M$ is bounded, show that $X + Y$ is bounded.*

Problem 5 (2 points)**.** *If $M \xrightarrow{f} N$ is a morphism of topological R-modules, show that $f(X)$ is a bounded subset of N.*

Problem 6 (2 points)**.** *Show that a finite union of power-bounded subsets of R is power-bounded.*

Problem 7 (2 points)**.** *Show that a bounded and topologically nilpotent subset of R is power-bounded.*

Problem 8 (2 points)**.** *Let X and Y be topologically nilpotent subsets of R. Show that XY is topologically nilpotent.*

Problem 9 (2 points)**.** *Let $X \subseteq R$ be power bounded and Y topologically nilpotent. Show that XY is topologically nilpotent.*

Solutions should be e-mailed to my institute e-mail address (my second name (franke) at math dot uni hyphen bonn dot de) before Monday December 9.