Conic Sections, 2D construction

See also Parabola, Ellipse, Hyperbola and their ATOs.

A cone of revolution (e.g., $\{(x, y, z); x^2+y^2 = m\cdot z^2\}$) is one of the simplest surfaces. Its intersections with planes are called conic sections. Apart from pairs of lines these conic sections are Parabolae, Ellipses or Hyperbolae. These curves have also other geometric definitions (e.g., The locus of points having the same distance from a focal point and a circle). See their Menu entries.

On the other hand, they are also more robust than these definitions show: Photographic images of conic sections are again conic sections; or in a completely different formulation: The intersection of a plane and any "quadratic cone", i.e.,
$\{(x, y, z) \mid a \cdot x^2 + b \cdot y^2 + c \cdot z^2 + d \cdot xy + e \cdot yz = 0\}$,
is **not** more complicated than planar sections of circular cones but are the same old Parabolae, Ellipses or Hyperbolae as above. A special case of this robustness is the fact that orthogonal projections of conic

sections in 3-space are again conic sections. This is illustrated in the program as follows:

Interpret the illustration as if it showed level lines on a hiking map. The equidistant parallel lines are the level lines of a sloping plane; the smaller the distance between these level lines the steeper the plane. The equidistant concentric circles are the level lines of a circular cone, as for example an ant lion would dig in sandy ground; without height numbers written next to the level lines we can of course not decide whether the circular level lines represent a conical mountain or a conical hole in the ground. We suggest that the blue level line and the vertex of the cone are at height zero and the other levels are higher up so that the cone is a hole.
The intersection curve between plane and cone has then an easy pointwise construction: Simply intersect level lines of the same height on the two surfaces. (These are lines with the same color in the program illustration.) This construction reveals a new geometric property of the intersection curve on the map, of this conic section:

Take the ratio of the distances from a point on the curve, (i) to the level line at height 0 of the plane (called *directrix*) and (ii) to the vertex at height zero of the cone (called *focus*). *This ratio is the same* as the ratio of adjacent level lines of plane and cone and therefore the same *for all points of this conic section.*