# Hyperbola

See also Parabola, Ellipse, Conic Sections and their ATOs.

The most common parametric equations for a Hyperbola with semi-axes aa and bb are:

$$x(t) = \pm aa \cosh(t), \quad y(t) = bb \sinh(t), \qquad t \in \mathbb{R};$$

and another version is:

$$x(t) = aa/\cos(t),\ y(t) = bb \sin(t)/\cos(t), \quad t \in [0, 2\pi].$$

The corresponding implicit equation is:

$$(x/aa)^2 - (y/bb)^2 = 1.$$

The function graphs: $\{(x, y);\ y = 1/x + m \cdot x\}$ are also Hyperbolae.

A geometric definition of the Hyperbola is:

> A Hyperbola is the set of points for which the **difference of the distance** from two focal points is constant.

Or:

> A Hyperbola is the set of points which have the **same distance** from a circle and a (focal) point **outside** that circle.

If one applies an inversion $(x, y) \rightarrow (x, y)/(x^2 + y^2)$ to a right Hyperbola (i.e. $aa = bb$) then one obtains a Lemniscate.

In the visualization of the complex map $z \rightarrow z + 1/z$ in Polar Coordinates, the image of the radial lines are the Hyperbolae:

$$x(R) = (R + 1/R) \cos \phi$$
$$y(R) = (R - 1/R) \sin \phi, \qquad R \in \mathbb{R}.$$

And the image of the standard Cartesian Grid under the complex map $z \rightarrow \sqrt{z}$ is a grid of two families of orthogonal Hyperbolae.

(H.K.)